A NEW TECHNIQUE FOR TIME-DOMAIN ULTRASONIC NDE OF EXTREMELY THIN PLATES

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INTRODUCTION

The strength of a joint is significantly affected by the thickness of the bond. Over the past quarter-century, a wide variety of ultrasonic techniques have been reported for the measurement of the thickness and wave velocity (or modulus) of a single layer. Among them are pulse-echo [1], resonance testing [2] and pulse interference [3] methods. Evidently as the plate thickness decreases the time interval between two successive echoes, Δt, decreases and finally the echoes become inseparable. All of the classical methods break down when the successive reflections from the two faces of the layer cannot be separated in the time-domain. In this paper a specimen will be called "thin" if $h < 3\ \lambda$, where λ is the wavelength in the interrogated material at the transducer center-frequency; conversely, it will be called "thick" if $h > 3\ \lambda$. In much of the aerospace applications, the typical adhesive thickness is of the order of 10^{-2} mm or 10^{-3} inch. In order to use any of the aforementioned methods, the transducer frequency would have to be larger than 150 MHz. By combining the theory of Fourier transforms with conventional ultrasonic hardware Kinra and Dayal developed a new ultrasonic NDE technique which removed this limitation [4]. Subsequently, this technique has been used for NDE of the properties of an extremely thin plate [5] as well as a three-layered medium (adherend-adhesive-adherend) where the combined thickness of the joint (h) qualifies as thin [6, 7].

A new time-domain ultrasonic NDE technique is reported in this paper for the measurement of the thickness of extremely thin plates. The problem is approached in two different ways. In the first approach, the normally-incident longitudinal wave transmitted through a plate is calculated as a sum of infinite series involving the incident field. In the second approach, by introducing the concept of a retrieve function (which is the inverse of the familiar transfer function) we show that the incident field can be expressed as a mere two-term summation over the transmitted field. This results in at least one order of magnitude saving in the computational time. The thickness is deduced from a root-sum-square comparison of the transmitted and the incident fields. A systematic

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sensitivity analysis of this technique has been carried out. This technique has been used to measure the thickness of aluminum plates with thickness ranging from 0.089 to 6.426 mm using (low frequency) 1-MHz transducers: $0.014 \le$ (thickness/wavelength) ≤ 1.0. The measurement error was found to be of the order of 1%.

THEORY

Consider an infinite plate of thickness h immersed in an elastic fluid (water) occupying the space $a \le x \le b$, $b=a+h$. Let $u^{inc}(x,t) = f(t-s_0x)$ be the incident wave traveling along the positive x direction, and $u^{trans}(x,t) = g^t(t-s_0x)$ be the transmitted wave, where t is time, s_0 is the slowness of water, and c_0 is its wavespeed ($s_0=1/c_0$).

Construction of the Transmitted Field from the Incident Field

A Lagrangian diagram indicating the space-time location of a wave front which occupied the position x=0 at time t=0 is shown in Fig.1. A plane wave, Ray 1, is normally incident on the plate. This results in an infinite series of reflected and transmitted pulses. The total transmitted field may be written as [8]

$$u^{trans}(x,t) = T_{12}T_{21}\sum_{m=0}^{M} R_{21}^{\ 2m} f\left(t-s_0x-h((2m+1)s-s_0)\right) , \qquad M \to \infty \tag{1}$$

where the transmission and reflection coefficients are now given, respectively, by $T_{12}=2z/(z+z_0)$ and $R_{12}=(z-z_0)/(z+z_0)$. The reflection coefficient $|R_{21}|$ is always less than one. Therefore the series in eq.(1) is guaranteed to converge. One needs to carry the summation from 0 to some large but finite number M. Suppose the maximum value of $|f(t)|$ is f_{max}, then the truncation error is [8]

$$E_m \le R_{12}^{\ 2(M+1)} f_{max} \tag{2}$$

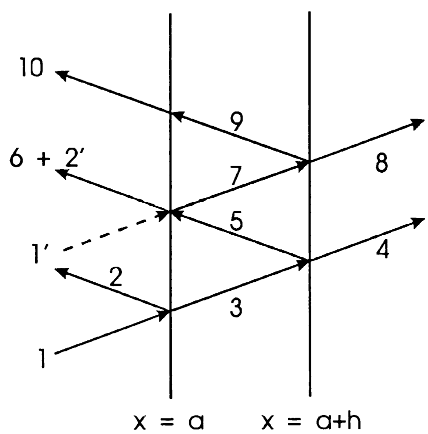

Fig.1 Various reflections from and transmissions through a plate.

From the practical point of view of the design of the experiment, one selects an acceptable level of the truncation error and uses eq.(2) to calculate M.

Construction of the Incident Field from the Transmitted Field

With $\eta = t - s_0 x$, let $F(\omega)$ and $G^t(\omega)$ be the Fourier transform (FT) of $f(\eta)$ and $g^t(\eta)$, respectively. The transfer function for a single plate is well known [4]

$$H^*(\omega) = \frac{G^t(\omega)}{F(\omega)} = \frac{T_{01}T_{10}e^{-i\omega(s-s_0)h}}{1-R_{01}{}^2 e^{-i2\omega sh}} \tag{3}$$

We note that the inverse Fourier transform (IFT) of $H^*(\omega)$ is the infinite series (1) with f( ) replaced by the Dirac-delta function $\delta(\,)$. We now introduce a retrieve function: $Q(\omega)=F(\omega)/G(\omega)$, which is merely the inverse of the transfer function. Let $q^t(\eta)$ be the IFT of $Q^t(\omega)$. It can be readily shown that [9]

$$q^t(\eta) = \frac{(2\pi)^{1/2}}{T_{01}T_{10}}\Big(\delta[\eta-(s_0-s)h]-R_{01}{}^2\delta[\eta-(s_0+s)h]\Big) \tag{4}$$

For later use, we define $q_1(t)=(2\pi)^{1/2}(T_{01}T_{10})^{-1}\delta[\eta-(s_0-s)h]$ and $q_2(t)=-(2\pi)^{1/2}(T_{01}T_{10})^{-1}R_{01}{}^2\delta[\eta-(s_0+s)h]$. Finally, given the transmitted field, the incident field can be calculated as a convolution of $q^t(\eta)$ and $g^t(\eta)$

$$f(\eta) = \frac{1}{T_{01}T_{10}}\Big(g^t[\eta-(s_0-s)h]-R_{01}{}^2 g^t[\eta-(s_0+s)h]\Big) \tag{5}$$

We now compare eqs.(1) and (5). In (1), an evaluation of $g^t(\eta)$ from $f(\eta)$ requires an infinite sum. In comparison, an evaluation of $f(\eta)$ requires the summation of only two terms involving the $g^t(\eta)$. This results in at least an order of magnitude saving in the computational effort to accomplish the same NDE objectives.

We now show that $q^t(\eta)$ is the excitation required to produce a Dirac-delta response. The retrieve function, $q^t(\eta)$, consists of two pulses: $q_1(t)$ and $q_2(t)$, see eq.(4). Let's consider the first pulse $q_1(t)=(2\pi)^{1/2}(T_{01}T_{10})^{-1}\delta[\eta-(s_0-s)h]$ normally incident along Ray 1. Upon through-transmission it produces Ray 4 given by $(2\pi)^{1/2}\delta(\eta-s_0h)$. After suffering a pair of internal reflections at x=a and x=b, Ray 7 is produced and is given by $(2\pi)^{1/2}T_{10}{}^{-1}R_{01}{}^2\delta[\eta-(s_0+s)h]$ whereupon it becomes clear that the second pulse, $q_2(t)$ (Ray 1' shown in Fig.1) of eq.(4), coincides with Ray 7. Indeed, upon transmission at x=a Ray 1' becomes $-(2\pi)^{1/2}T_{10}{}^{-1}R_{01}{}^2\delta[\eta-(s_0+s)h]$ which exactly cancels Ray 7. This is the main point of this discussion: once Ray 7 is canceled, there are no further internal reverberations in the plate and $\delta(\eta-s_0h)$ comprises the entire transmitted field. Accordingly, $q^t(\eta)$ may be called the impulse excitation function.

NUMERICAL PROCEDURE AND SENSITIVITY ANALYSIS

Let £(t;h) (£=g, f) be the reconstructed field. and £*(t) be the corresponding measured one. A root-mean-square error function is introduced to quantify the difference between the theory and the experiment,

$$E_{£}(h) = \frac{1}{£*_{max}} \left[\frac{1}{N} \sum_{i=1}^{N} (£(t_i;\ h) - £^*(t_i))^2 \right]^{\frac{1}{2}} \tag{6}$$

where $£*_{max}$=MAX|$£*(t)$|, N is the total number of points at which $£^*(t_i)$ is measured. Eq.(6) was used to estimate the thickness of a plate in this research. From the viewpoint of the Inverse Problem, it is critically important that £(t;h) be very sensitive to h.A time-averaged sensitivity of £(t,h) to h is defined as

$$S_{£,h} = \left[\frac{1}{T} \int_0^T \left(\frac{h}{£_{max}} \frac{\partial £}{\partial h}\right)^2 dt \right]^{\frac{1}{2}} \tag{7}$$

Moreover, in eq.(6), by using a Taylor series expansion of £(t,h) about h_{TRUE} (i.e. $h=h_{TRUE}+\Delta h$) and retaining only the terms linear in Δh, it can be readily shown (as expected) that $S_{£,h} = E_{£} / (\Delta h/h_{TRUE})$. Finally, in the present case of discrete data,

$$S_{£,h} = \frac{1}{£_{max}} \left[\frac{1}{N} \sum_{i=1}^{N} \left(£(t_i\ ,\ h+\Delta h) - £(t_i\ ,\ h)\right)^2 \right]^{\frac{1}{2}} / (\Delta h/h_{TRUE}) \tag{8}$$

Typically, we used $\Delta h/h_{TRUE}=10^{-2}$ in all the numerical work. $S_{£,h}$ versus h computed by the use of equation (8) is plotted in Fig.2. Note that $S_{£,h}$ increases with h. If the sensitivity $S_{£,h}$ = 0, we lose the only equation we had to estimate h (when plotted against Δh, $E_{£}=0$). Conversely, if $S_{£,h}$ is high, a small change in h causes a large change in $E_{£}$ and one can deduce h_{TRUE} very accurately.

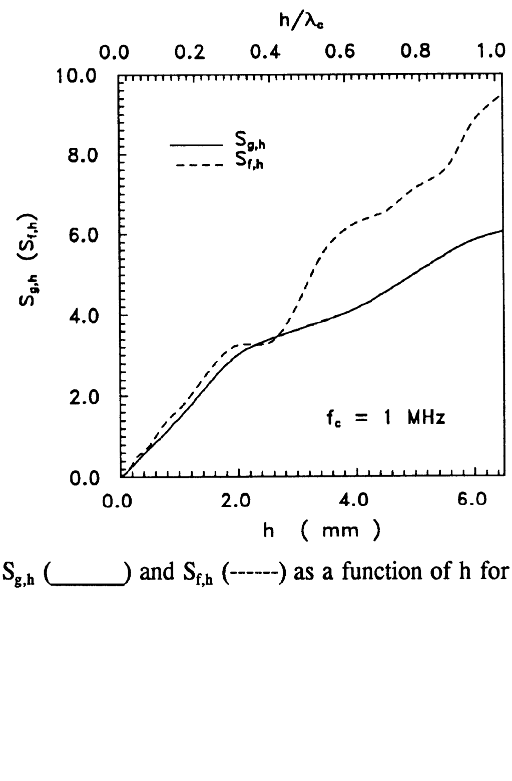

Fig.2 Sensitivity, $S_{g,h}$ (______) and $S_{f,h}$ (------) as a function of h for an aluminum plate.

EXPERIMENTAL PROCEDURES

A pair of accurately matched, broadband, videoscan, water immersion, piezoelectric transducers with a center frequency of 1 MHz was used for generating and receiving the elastic waves. An experiment was initiated at time $t=0$ by a triggering pulse produced by a Tektronix PG501 Pulse Generator. This pulse was used to trigger a Wavetek 190 Function Generator. The function generator produces a 1 μs sinusoidal pulse which is amplified with the help of an ENI A150 Power Amplifier to about 150 volts peak-to-peak amplitude and applied to the transmitting transducer. Simultaneously, the amplified signal was used to trigger the oscilloscope to minimize the system jitter. The received signal was sent to the oscilloscope and digitized at a sampling interval of $\Delta t \leq 10$ ns. To reduce random errors, each signal was averaged 64 times. A computer was used to control all the operations of the digital oscilloscope through an IEEE 488 bus. The digitized information was sent to the computer for further analysis. Each thickness measurement took about 2 minutes on a NEC 486 computer [8].

RESULTS AND DISCUSSION

All NDE proceeds in two stages. (I) The Forward Problem: in the present paper by forward problem we mean given the plate thickness compare the measured signal, $£^*(t)$, and the theoretically predicted (reconstructed) signal, $£(t)$. (II) The Inverse Problem: by inverse problem we mean deduce h from a comparison of $£(t,h)$ and $£^*(t)$.

The Forward Problem

Ten aluminum plates with thickness ranging from 0.089 mm to 6.426 mm were tested with 1 MHz transducer; see Table I. For three typical values of h, namely, h=0.089, 0.254 and 1.003 mm g(t) and $g^*(t)$ are compared in Fig.3. The comparison is considered excellent. Fig.4 shows the corresponding comparisons of f(t) and $f^*(t)$ for the

Table I Thickness Measurement Data

$h_c(\pm 3*10^{-3}$ mm)	h^g_{NDE}(mm)	$Error^g(\mu m)$	$Error^g\%$	h^f_{NDE}(mm)	$Error^f(\mu m)$	$Error^f\%$	h_c/λ_c
0.089	0.089	0	0.0	0.089	0.0	0.0	0.014
0.254	0.254	0	0.0	0.255	1	0.4	0.04
0.521	0.520	1	0.1	0.526	5	1.0	0.082
0.635	0.640	5	0.8	NA	NA	NA	0.1
1.003	1.010	7	0.7	1.000	3	0.3	0.158
1.486	1.480	6	0.4	1.476	10	0.7	0.234
2.276	NA	NA	NA	2.259	17	0.7	0.358
3.175	3.050	125	4.0	3.169	6	0.2	0.5
6.325	6.380	55	0.9	NA	NA	NA	1.0
6.426	NA	NA	NA	6.35	76	1.2	1.0

NA: Not available

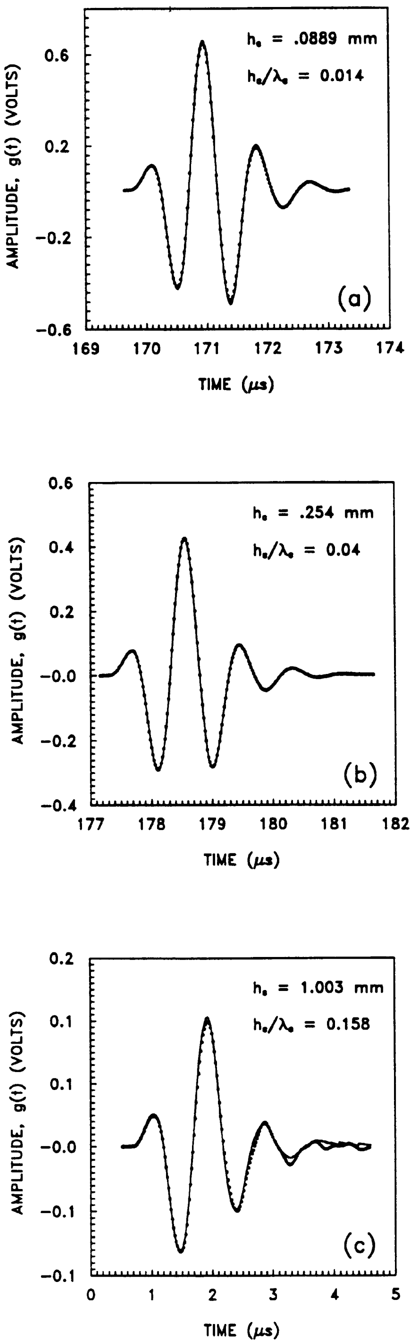

Fig.3 Comparison of the reconstructed signal, g(t)(______), and the measured signal, $g^{*}(t)$(......).

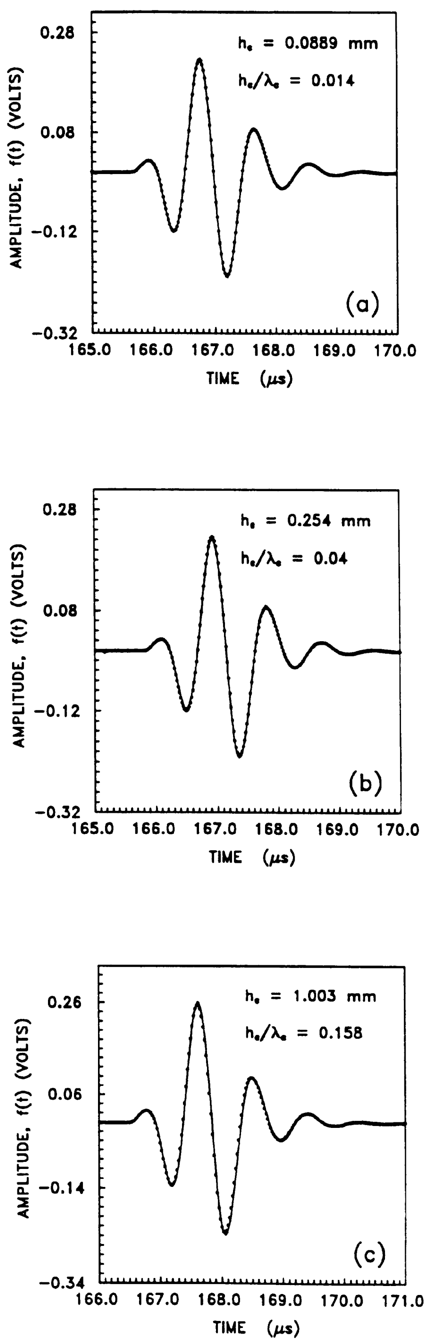

Fig.4 Comparison of the reconsted signal, f(t)(______), and the measured signal, $f^{*}(t)$(......).

same three plates. In all cases, $E_£$ was found tobe less than 1%. The values of (h/λ) ranges from 0.014 to 1 is also shown in Table 1.

The Inverse Problem

The normalized error function $E_£$, eq.(6), is plotted against h in Fig.5. The plate thickness was measured with a pair of calipers: h_c=0.245 mm ±2.54 μm $(10\pm0.1)10^{-3}$ inch. The numerical search is carried over $0.0<h<0.50$ mm, i.e. $h_c\pm100\%$. As expected, $E_£$ goes through a well-defined minimum in the vicinity of h_c. This yields the "best estimate of the true value of thickness", $h^£_{NDE}$. That $E_£$ is not zero at the minimum may be attribute to the random error in $£^*(t)$. The slope of the two (nearby) straight lines is the sensitivity, $S_{£,h}$. For thick plates, $S_{£,h}$ is large and the minimum can be located with a higher precision; for thin plates, $S_{£,h}$ is small, one observe a shallow $E_£$ curve and the location of the minimum becomes proportionally imprecise. Fig.5 typifies all sub-half-wavelength plate, i.e. $(h/\lambda) < 0.5$.

Ten aluminum plates were tested and the inverse algorithm was applied to them. The results are presented in Table I. The measured error is defined as $\epsilon=(h_{NDE}-h_c)/h_c$, where h_c is taken to be the "true value" even though it suffers from a measurement error of ±2.54 μm or 10^{-4} inch. The typical error is of the order of 1%. We observe that the new technique works very well for extremely thin plates, $(h/\lambda)=O(10^{-2})$. With the exception of h_c/λ in the vicinity of 0.5, the error $\epsilon<1$ %. The reason for the slightly large error (4%) for the case of h_c=3.175 mm is not well understood. One conjecture is that at h_c=3.175 mm $(h_c/\lambda=0.5)$, the second shear resonance was excited inside the specimen. Such resonance could result in large error.

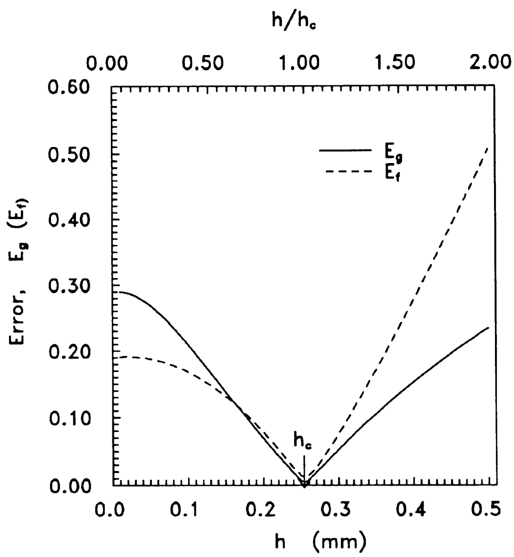

Fig.5 Plot of error function, E_g (E_f) (h_c=0.254 mm, h/λ=0.04).

CONCLUSION

A new NDE technique has been developed to determine the thickness of an extremely thin (sub-wavelength) plate specimen using only time-domain information. Aluminum plates ranging in thickness from 0.089 to 6.426 mm were tested using 1 MHz transducer: $0.014 < (h_c/\lambda) < 1.0$. The error was found to be 1% for h/λ down to 10^{-2}. In dimensional terms, plates with a thickness of 100 μm can be measured with an accuracy of ± 1 μm using low-frequency, 1-MHz transducers.

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